(**A**) $\frac{1}{2}$ (**B**) 1 (**C**) $\frac{3}{2}$ (**D**) 2 (**E**) $\frac{5}{2}$

Solution: Any drawn triangle has the width of at least 1 and the height of at least 1. Therefore the area of any triangle is at least $\frac{1}{2} \cdot 1 \cdot 1 = 1/2$. The example is obtained by connecting the three highest marked points in the picture.

4 points

**# 11.** Helen wants to spend 18 consecutive days visiting her Grandma. Her Grandma reads her story books on story days Tuesday, Saturday and Sunday. Helen wants to spend as many story days with her Grandma as possible. On which day of the week should she start her visit?

(**A**) Monday (**B**) Tuesday (**C**) Friday (**D**) Saturday (**E**) Sunday

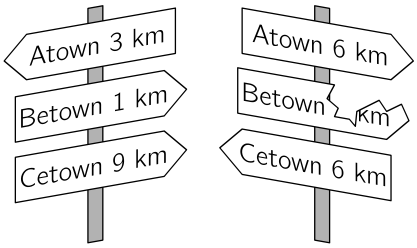
Solution: A seven day period is, of course periodic, so concentrate on the 18-14=4 remaining days. A start on Saturday contains all story days, so it is maximum.

**# 12.** The integers $a$, $b$, $c$ and $d$ satisfy $ab = 2cd$. Which of the following numbers could not be the value of the product $abcd$?

(**A**) 50 (**B**) 100 (**C**) 200 (**D**) 450 (**E**) 800

Solution: Condition gives $abcd = 2(cd)^2$ (twice a perfect square). Of the numbers given the only one not twice a perfect square is 100. Note that we can find examples to show that all other answers are possible.

**# 13.** The shortest path from Atown to Cetown runs through Betown. Walking on this path from Atown to Cetown, we would first find the signpost shown on the left. Later we would find the signpost shown on the right. What distance was written on the broken sign?




(**A**) 1 km (**B**) 2 km (**C**) 3 km (**D**) 4 km (**E**) 5 km

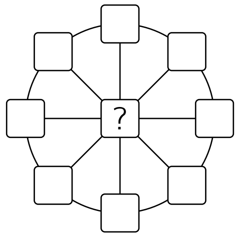
Solution: From both signposts you get the information, that the distance between Atown and Cetown is 12 km. The left signpost shows that the distance between Atown and Betown is 4 km. So the distance between Betown and Cetown is 8 km and the missing distance is 2 km.

**# 14.** An isosceles triangle has a side of length 20 cm. Of the other two side lengths, one is equal to 2/5 of the other. Which of the following values is the perimeter of this triangle?

(**A**) 36 cm (**B**) 48 cm (**C**) 60 cm (**D**) 90 cm (**E**) 120 cm

Solution: To fulfill the property of being isosceles and the given equation there are two possibilities: 20 cm, 8 cm, 20 cm and 20 cm, 20 cm, 50 cm. The second does not fulfill the triangle inequality. So the perimeter must be 48 cm.

**# 15.** Tom wants to write a number in each of the nine cells of the figure shown. He wants the sum of the three numbers on each diameter to be 13 and the sum of the eight numbers on the circumference to be 40. What number has Tom to write in the central cell?




(<u>**A**</u>) 3 (**B**) 5 (**C**) 8 (**D**) 10 (**E**) 12

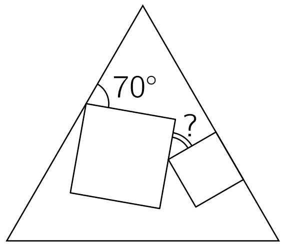
SOLUTION: Adding 4 times the diameter we get 52 where the central cell $n$ is added 4 times and each cell of the circumference only once. So $4 \cdot n = 52 - 40 = 12$. Hence $n = 3$.

**# 16.** Masha put a multiplication sign between the $2^{\text{nd}}$ and $3^{\text{rd}}$ digits of the number 2020 and noted that the resulting product $20 \cdot 20$ is a square number. How many numbers between 2010 and 2099 (including 2020) have the same property?

(**A**) 1 (**B**) 2 (<u>**C**</u>) 3 (**D**) 4 (**E**) 5

SOLUTION: Solutions are $20 \cdot 20$, $20 \cdot 45$, $20 \cdot 80$. If the 4 digits of the year are 2, 0, $a$ and $b$, then we get $20 \cdot \overline{ab} = n^2$ for some positive integer $n$. Because $20 = 5 \cdot 2^2$, $\overline{ab} = 5 \cdot m^2$ for some positive integer $m$. As $10 \leq \overline{ab} \leq 99$, $m = 2$, $m = 3$ or $m = 4$ are possible.

**# 17.** Two squares of different size are drawn inside an equilateral triangle. One side of one of these squares lies on one of the sides of the triangle, as shown. What is the size of the angle marked by the question mark?




(**A**) 25° (**B**) 30° (**C**) 35° (**D**) 45° (<u>**E**</u>) 50°

SOLUTION: The sum of the angles of the pentagon at the top of the figure is 540°. The known angles of this pentagon are 70°, 60°, 90° and 270°. So the missed angle is $540° - 490° = 50°$.

**# 18.** Luca began a 520 km trip by car with 14 litres of fuel in the car tank. His car consumes 1 litre of fuel per 10 km. After driving 55 km, he reads a road sign showing the distances from that point to five petrol stations ahead on the road. These distances are 35 km, 45 km, 55 km, 75 km and 95 km. The capacity of the car's fuel tank is 40 litres and Luca wants to stop just once to fill the tank. How far is the petrol station that he should stop at?

(**A**) 35 km (**B**) 45 km (**C**) 55 km (<u>**D**</u>) 75 km (**E**) 95 km

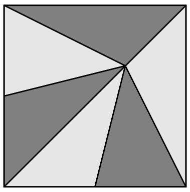
SOLUTION: When Luca reads the information plate he can still drive 85 km at most with the fuel in his tank and has still 465 km to go. So he can't reach the fifth station. As a full tank takes him at most 400 km he should not stop within the next 65 km. So only the fourth station is possible.

**# 19.** Let $17x + 51y = 102$. What is the value of $9x + 27y$ ?

(<u>**A**</u>) 54 (**B**) 36 (**C**) 34 (**D**) 18
(**E**) The value is undetermined.

SOLUTION: If we divide the given equation by 17 we get $x + 3y = 6$. If we multiply this by 9 we get $9x + 27y = 54$.

**# 20.** A square shaped stained glass window of 81 dm$^2$ is made out of six triangles of equal area (see figure). A fly is sitting exactly on the spot where the six trangles meet. How far from the bottom of the window is the fly sitting?



(**A**) 3 dm (**B**) 5 dm (**C**) 5.5 dm (**<u>D</u>**) 6 dm (**E**) 7.5 dm

SOLUTION: The triangle on the upper side of the window has an area of 1/6 of the whole window. So its height is equal to 1/3 of the height of the window which is 9 dm. So the fly is sitting 6 dm above the bottom.

5 points

**# 21.** The digits from 1 to 9 are randomly arranged to make a 9-digit number. What is the probability that the resulting number is divisible by 18?

(**A**) $\frac{1}{2}$ (**<u>B</u>**) $\frac{4}{9}$ (**C**) $\frac{5}{9}$ (**D**) $\frac{1}{3}$ (**E**) $\frac{3}{4}$

SOLUTION: All such numbers are divisible by 9 because their digit sum is 45. So the last digit must be even, hence the probability is $\frac{4}{9}$.

**# 22.** A hare and a tortoise competed in a 5 km race along a straight line. The hare is five times faster than the tortoise. The hare mistakenly started perpendicular to the route. After a while he realized his mistake, then turned and ran straight to the finish point. He arrived at the same time as the tortoise. What is the distance between the hare's turning point and the finish point?

(**A**) 11 km (**B**) 12 km (**<u>C</u>**) 13 km (**D**) 14 km (**E**) 15 km

SOLUTION: Denote the strating point $S$, the turning point $T$ and the finish point $F$.

Then we have $\|SF\| = 5$ km, $\|TF\| = x$ km and $\|ST\| = 25 - x$ km since arriving at the same time means that the hare covered the distance of 25 km. The Pythogorean theorem implies $5^2 + (25 - x)^2 = x^2$ and we get $x = 13$ km.

**# 23.** There are some squares and triangles on the table. Some of them are blue and the rest are red. Some of these figures are large and the rest are small. We know the following two facts are true:

1) if the figure is large then it is a square and

2) if the figure is blue then it is a triangle.

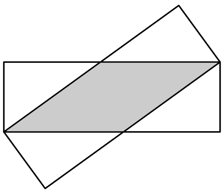
Which of the statements A–E must be true?

(**A**) All red figures are squares. (**B**) All squares are large. (**C**) All small figures are blue.

(**D**) All triangles are blue. (**<u>E</u>**) All blue figures are small.

SOLUTION: There might be red triangles, hence A and D may not be true. There might be small squares, hence B may not be true. There might be small figures that are red, hence C may not be true. E must be true because every blue figure is a triangle, and every large figure is a square, so every blue figure is small.

**# 24.** Two identical rectangles with sides of length 3 cm and 9 cm overlap, as shown in the diagram.



What is the area of the overlap of the two rectangles?

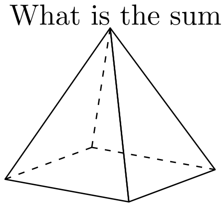
(**A**) 12 $cm^2$ (**B**) 13.5 $cm^2$ (**C**) 14 $cm^2$ (**D**) 15 $cm^2$ (**E**) 16 $cm^2$

SOLUTION: Because of the symmetry horizontal unshaded side and longer skew part are equal. We denote the length of these segments by $x$.

Then the length of the shaded horizontal side is $9-x$. The Pythogorean theorem gets us $3^2+x^2=(9-x)^2$, so $x=4$.

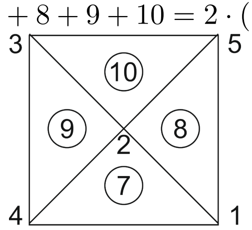
The shaded area is $27-3\cdot 4=15$.

**# 25.** Kanga labelled the vertices of the square-based pyramid using 1, 2, 3, 4 and 5 once each. For each face Kanga calculated the sum of the numbers on its vertices. Four of these sums are 7, 8, 9 and 10. What is the sum of the numbers at the the vertices of the fifth face?



(**A**) 11 (**B**) 12 (**C**) 13 (**D**) 14 (**E**) 15

SOLUTION: The sum of numbers at the vertices of the base is at least $1+2+3+4=10$. If it equals 10 then the top of the pyramid is marked with 5 and all four sums are at least $5+1+2=8>7$ — a contradiction. Hence the sums 7, 8, 9 and 10 are calculated for the side faces. Therefore if the top vertex is marked with $x$ then $7+8+9+10=2\cdot(1+2+3+4+5)+2x$. Hence $x=2$ and the fifth sum equals $1+3+4+5=13$.




**# 26.** A large cube is built using 64 smaller identical cubes. Three of the faces of the large cube are painted. What is the maximum possible number of small cubes that have exactly one face painted?

(**A**) 27 (**B**) 28 (**C**) 32 (**D**) 34 (**E**) 40

SOLUTION: There are only two possibilities: 3 faces around the corner or U-shape that is two opposite faces and one in between.

In the first case we have 27 small cubes with exactly one face painted (three times $3\times 3$).

In the second case we have 32 small cubes (two times $3\times 4$ plus $2\times 4$).

**# 27.** Anna wants to write a number in each of the squares of the grid so that the sum of the four numbers in each row and the sum of the four numbers in each column are the same. Se has already written some numbers, as shown. What number does she write in the shaded square?

| | | | |
|---|---|---|---|
| 1 | | 6 | 3 |
| | 2 | 2 | 8 |
| | 7 | | 4 |
| | | 7 | |

(**A**) 5 (**B**) 6 (**C**) 7 (**D**) 8 (**E**) 9

SOLUTION: A formal way: denote the number in the bottom right corner by $x$ then the sum must be $x + 15$ and fill out the table.

A nice way: the first line and the second column have an empty cell in common so the cell in the 4th line and the second column must contain 1.

Now the last line and the last column have the bottom right corner in common so we have to have 7 in the bottom left corner.

**# 28.** Alice, Belle and Cathy had an arm-wrestling contest. In each game two girls wrestled, while the third rested. After each game, the winner played the next game against the girl who had rested. In total, Alice played 10 times, Belle played 15 times and Cathy played 17 times. Who lost the second game?

(**A**) Alice

(**B**) Belle

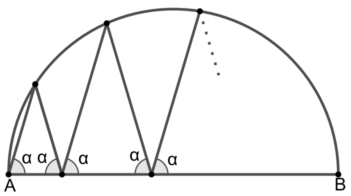
(**C**) Cathy

(**D**) either Alice or Belle could have lost the second game

(**E**) either Belle or Cathy could have lost the second game

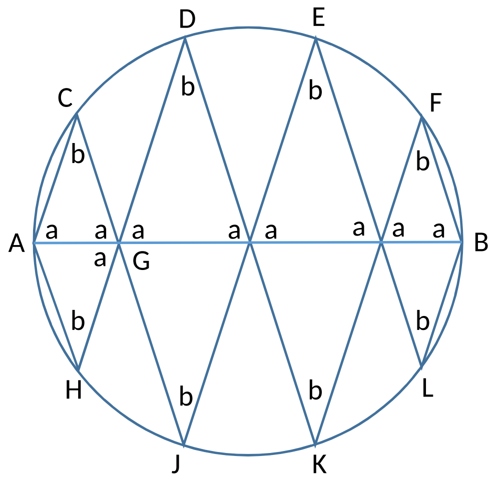
SOLUTION: There were $\frac{10+15+17}{2} = 21$ games. Since Alice was resting during $21 - 10 = 11$ games and nobody gets to rest more than one game in a row, Alice rested during the first, third, fifth, ... games. Therefore, she lost the second game. Such a scenario is possible: First nine games could be BC AB BC AB BC AB BC AB BC (that is, alternating BC AB) and the other 12 alternating AC BC.

**# 29.** A zig-zag line starts at the point $A$, at one end of the diameter $AB$ of a circle. Each of the angles between the zig-zag line and the diameter $AB$ is equal to $\alpha$ as shown. After four peaks, the zig-zag line ends at the point $B$. What is the size of angle $\alpha$?




(**A**) 60° (**B**) 72° (**C**) 75° (**D**) 80° (**E**) Another answer

SOLUTION: Reflect the figure along its diametre, so that $arc(AC) = arc(AH), arc(CD) = arc(HJ)$, etc. Note that D,G,H are collinear because of the equality of alternate angles (both $a$). Also note that $AC$ and $GD$ (and so $HD$) are parallel. It follows that $arc(CD) = arc(AH)$. In particular, we now have $arc(AC) = arc(AH) = arc(CD) = arc(HJ)$. Repeating the argument we in fact have (by symmetry or parallel lines) that all arcs in the figure are equal, namely $arc(AC) = ... = arc(BL)$. So the decagon $ACDEFBLKJH$ is regular. In particular $a = 144^o/2 = 72^o$. // Other solution: For reasons of symmetry it is clear that the zig-zag line goes through the centre M of the circle. Because they are all isoscele triangles with the same angle $\alpha$ at their base, $\triangle AGC$, $\triangle GMD$ and $\triangle CAM$ are similar to each other and $\triangle GMD$ and $\triangle CAM$ are congruent because they both have the radius of the circle as their longer side. Because of $arc(AC) = arc(CD)$ (see above) the $\triangle DCM$ is congruent too. So two times the angle $\beta$ is equal to angle $\alpha$ and we get $5 * \beta = 180^o$ or $\alpha = 72^o$.




**# 30.** Eight consecutive three-digit positive integers have the following property: each of them is divisible by its last digit. What is the sum of the digits of the smallest of the eight integers?

(**A**) 10 (**B**) 11 (**C**) 12 (<u>**D**</u>) 13 (**E**) 14

SOLUTION: The set of the last digits can only be $1, 2, \dots, 8$ or $2, 3, \dots, 8, 9$.

If $\overline{abc}$ is divisible by $c$, then $\overline{abc} - c$ is divisible by 10 so $\overline{ab0}$ is divisible by $c$ for all $c$ in the sets mentioned in the first line. It follows that $\overline{ab0}$ must be divisible by 2, 3, 5, 7 and 8. So it is divisible by 840. So the smallest number $\overline{abc}$ must be 841.

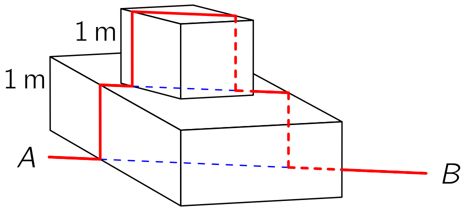
3 points

**# 1.** What is the sum of the last two digits of the product $1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$?

(**A**) 2 (**B**) 4 (**C**) 6 (**D**) 8 (**E**) 16

SOLUTION:

**# 2.** An ant walked every day on a straight horizontal line path from $A$ to $B$, which are 5 m apart. One day humans placed on its path two strange obstacles of height 1 m each. Now the ant walks along or above the same straight line except that it now has to climb up and down vertically over both the two obstacles, as in the picture. How long is its path now?

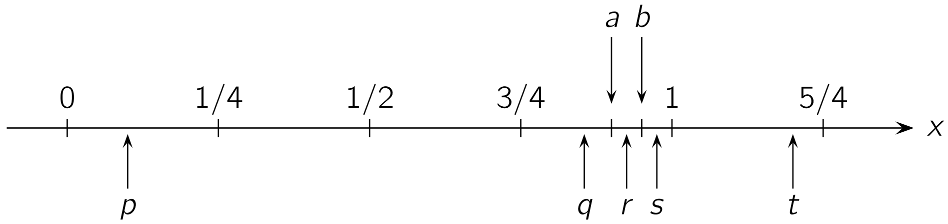



(**A**) 7 m (**B**) 9 m (**C**) $5 + 4\sqrt{2}$ m (**D**) $9 - 2\sqrt{2}$ m

(**E**) the length depends on the angles the obstacles are situated along the path

SOLUTION: The horizontal part of the path is exactly as long as the original path AB. The can be seen by projecting the new path onto the horizontal plane. The extra length comes from the four vertical parts of the path. The total length is 5+(1+1+1+1)=9 m.

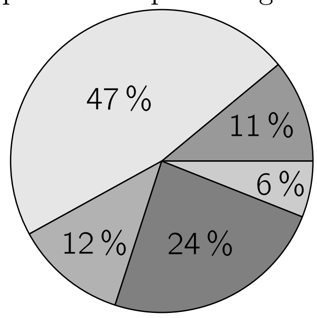
**# 3.** Rene marked two points $a$ and $b$ as accurately as possible on the number line. Which of the points $p$, $q$, $r$, $s$, $t$ on the number line best represents their product $ab$?




(**A**) $p$ (**B**) $q$ (**C**) $r$ (**D**) $s$ (**E**) $t$

SOLUTION: Both $a$ and $b$ are less than 1 so their product is less than 1. In fact $ab < a$ which excludes points $r$, $s$, $t$. Also certainly $a, b > \frac{1}{2}$ so $ab > \frac{1}{4}$so certainly $p$ is excluded. This leaves $q$ which, moreover, is realistically placed because $b \approx 1$ so $ab \approx a$, so $ab$ is close but less that $a$. Such is the case as with the location of $q$.

**# 4.** The pie chart shows how the students of my school get to school. Approximately twice as many go by bike as use public transport and roughly the same number come by car as walk. The rest use a moped. What percentage use a moped?




(**A**) 6 % (**B**) 11 % (**C**) 12 % (**D**) 24 % (**E**) 47 %

SOLUTION:

**# 5.** The sum of five three-digit numbers is 2664, as shown on the board. What is the value of $A+B+C+D+E$?

$$\begin{array}{r} \mathsf{A\,B\,C} \\ +\,\mathsf{B\,C\,D} \\ +\,\mathsf{C\,D\,E} \\ +\,\mathsf{D\,E\,A} \\ +\,\mathsf{E\,A\,B} \\ \hline \mathsf{2\,6\,6\,4} \end{array}$$

(**A**) 4 (**B**) 14 (**C**) 24 (**D**) 34 (**E**) 44

SOLUTION: $(100A+10B+C)+(100B+10C+D)+(100C+10D+E)+(100D+10E+A)+(100E+10A+B) = 111(A+B+C+D) = 2664$ so $A+B+C+D = 2664 : 111 = 24$.

**# 6.** What is the value of $\dfrac{1010^2+2020^2+3030^2}{2020}$?

(**A**) 2020 (**B**) 3030 (**C**) 4040 (**D**) 6060 (**E**) 7070

SOLUTION: $\dfrac{1010^2+2^2\cdot 1010^2+3^2\cdot 1010^2}{2\cdot 1010} = \dfrac{(1+2^2+3^2)1010^2}{2\cdot 1010}$.

**# 7.** Let $a$, $b$ and $c$ be integers satisfying $1 \leqslant a \leqslant b \leqslant c$ and $abc = 1\,000\,000$. What is the largest possible value of $b$?

(**A**) 100 (**B**) 250 (**C**) 500 (**D**) 1000 (**E**) 2000

SOLUTION:

**# 8.** If $D$ dogs weigh $K$ kilograms and $E$ elephants weigh the same as $M$ dogs, how many kilograms does one elephant weigh?

(**A**) $DKEM$ (**B**) $\frac{DK}{EM}$ (**C**) $\frac{KE}{DM}$ (**D**) $\frac{KM}{DE}$ (**E**) $\frac{DM}{KE}$

SOLUTION:

**# 9.** There are two dice. Each one has two red faces, two blue faces and two white faces. If we roll both dice together, what is the probability that both show the same color?

(**A**) $\frac{1}{12}$ (**B**) $\frac{1}{9}$ (**C**) $\frac{1}{6}$ (**D**) $\frac{2}{9}$ (**E**) $\frac{1}{3}$

SOLUTION:

**# 10.** Which of the following numbers is not divisible by 3 for any integer $n$?

(**A**) $5n+1$ (**B**) $n^2$ (**C**) $n(n+1)$ (**D**) $6n-1$ (**E**) $n^3-2$

SOLUTION:

4 points

**# 11.** A blue rectangle and a red rectangle are overlapping. The figure shows 4 different such cases. We denote by $B$ the area of the part of the blue rectangle that is not common to the two rectangles, and we denote by $R$ the area of the red rectangle that is not common to the two. Which of the following statements is true about the quantity $B-R$?